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Parton distribution functions and quark orbital motion

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Abstract. A covariant version of the quark–parton model is studied. The dependence of the structure functions and parton distributions on the 3D intrinsic motion of the quarks is discussed. The important role of the orbital momentum of the quark, which is a particular case of intrinsic motion, appears as a direct consequence of the covariant description. The effect of the orbital motion is substantial, especially for polarized structure functions. At the same time, the procedure for obtaining the momentum distributions of polarized quarks from the combination of polarized and unpolarized structure functions is suggested.

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1 Introduction

The nucleon structure functions are a basic tool for understanding the internal structure of the nucleon in the language of QCD. At the same time, measurement and analysis of the structure functions represent an important experimental test of this theory. Unpolarized nucleon structure functions are known with high accuracy in a very broad kinematical region, but in recent years also some precision measurements on the polarized structure functions have been completed [1–8]. For the present status of the spin structure of the nucleon, see e.g. [9] and references therein. The more formal aspects of the structure functions of the nucleon are explained in [10]. In fact, only the complete set of the four electromagnetic unpolarized and polarized structure functions F_1, F_2, g_1 and g_2 can give a consistent picture of the nucleon. However, this picture is usually drawn in terms of the distribution functions, which are connected with the structure functions in some model-dependent way. Distribution functions are not directly accessible from experiment, and the model that is normally applied for their extraction from the structure functions is the well known quark–parton model (QPM). Application of this model to analysis and interpretation of the unpolarized data does not create any contradiction. On the other hand, the situation is much less clear in the case of the spin functions g_1 and g_2.

In our previous study [12, 13] we have suggested that a reasonable explanation of the experimentally measured spin functions g_1 and g_2 is possible in terms of a generalized covariant QPM in which the intrinsic motion of the quarks (i.e. 3D motion with respect to the nucleon rest frame) is consistently taken into account. Therefore the transversal momentum of the quarks appears in this approach on the same level as the longitudinal one. The quarks are represented by free Dirac spinors, which allows one to obtain an exact and covariant solution for the relations between the quark momentum distribution functions and the structure functions accessible from experiment. In this way the model (in its present LO version) contains no dynamics but only the "exact" kinematics of the quarks, so it can be an effective tool for analysis and interpretation of the experimental data on the structure functions, particularly for separating the effects of the dynamics (QCD) from the effects of the kinematics. This point of view is well supported by our previous results.

In the cited papers we showed that the model simply implies the well known sum rules (due to Wanzura–Wilczek, Efremov–Leader–Teryaev and Burkhardt–Cottingham) for the spin functions g_1, g_2. Simultaneously, we showed that the same set of assumptions implies a rather substantial dependence of the first moment Γ_1 of the function g_1 on the kinematical effects. Further, we showed that the model allows one to calculate the functions g_1 and g_2 from the unpolarized valence quark distributions, and the result is quite compatible with the experimental data. In [14] we showed that the model allows one to relate the transversity distribution to some other structure functions.

These results cannot be obtained from the standard versions of the QPM (naive or QCD improved), which are currently used for the analysis of experimental data on the structure functions. The reason is that the standard QPM is based on the simplified and non-covariant kinematics in the infinite momentum frame (IMF), which does not allow

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one to properly take into account the intrinsic or orbital motion of the quarks.

The subject of our previous study was the question: what is the dependence of the structure functions on the intrinsic motion of the quarks? The aim of the present paper is a discussion of related problems: how can one extract information about the intrinsic motion of the quarks from the experimentally measured structure functions? What is the role of the orbital momentum of the quarks, which is a particular case of the intrinsic motion?

The paper is organized as follows. In the first part of Sect. 2 the basic formulas, which follow from the generalized QPM, are presented. The general covariant relations are compared with their limiting case, which is represented by the standard formulation of the QPM in the IMF. In the next part of the section we calculate the 3D momentum distributions of the quarks and the structure functions are used as the input. The momentum distributions of positively and negatively polarized quarks are separately obtained from the combination of the structure functions F_2 and g_1 or the corresponding parton distributions $q(x)$ and $\Delta q(x)$. A particular form of intrinsic motion of the quarks is the orbital momentum. In Sect. 3 the role of the orbital momentum of the quark in covariant description is discussed, and it is shown why its contribution to the total angular momentum of the quark can be quite substantial. It is demonstrated that the orbital motion is an inseparable part of the covariant approach. The last section is devoted to a short summary and to our conclusions. In fact, this paper is inspired by many previous papers, see e.g. [15–27], in which the problem of the orbital momentum of the quarks in the context of nucleon spin was recognized and studied.

2 Structure functions and intrinsic quark motion

In our previous study [11–13] of the proton structure functions we showed how these functions depend on the intrinsic motion of the quarks. The quarks in the suggested model are represented by free fermions, which are in the rest frame of the nucleon described by a set of distribution functions with spheric symmetry, $G_k^{\pm}(p_0)\,\mathrm{d}^3p$, where $p_0=\sqrt{m^2+\mathbf{p}^2}$, and the symbol k represents the quark and antiquark flavors. These distributions measure the probability to find a quark of given flavor in the state

$$u(\mathbf{p},\lambda\mathbf{n})=\frac{1}{\sqrt{N}}\begin{pmatrix}\phi_{\lambda\mathbf{n}}\\ \frac{\mathbf{p}\sigma}{p_0+m}\phi_{\lambda\mathbf{n}}\end{pmatrix};$$
$$\frac{1}{2}\mathbf{n}\sigma\phi_{\lambda\mathbf{n}}=\lambda\phi_{\lambda\mathbf{n}}\,,\qquad N=\frac{2p_0}{p_0+m}\,,\tag{1}$$

where m and p are the quark mass and momentum, $\lambda=\pm1/2$, $\phi^{\dagger}_{\lambda\mathbf{n}}\phi_{\lambda\mathbf{n}}=1$ and $\mathbf{n}$ coincides with the direction of the polarization of the nucleon. The distributions together with the corresponding quark (and antiquark) charges e_k allow one to define the generic functions G and ΔG[1]:

$$G(p_0)=\sum_k e_k^2G_k(p_0)\,,\qquad G_k(p_0)\equiv G_k^+(p_0)+G_k^-(p_0)\,,\tag{2}$$

$$\Delta G(p_0)=\sum_k e_k^2\Delta G_k(p_0)\,,\quad \Delta G_k(p_0)\equiv G_k^+(p_0)-G_k^-(p_0)\,,\tag{3}$$

from which the structure functions can be obtained. If q is the momentum of the photon absorbed by the nucleon of momentum P and mass M, in which the phase space of the quarks is controlled by the distributions $G_k^{\pm}(p_0)\,\mathrm{d}^3p$, then there are the following representations of the corresponding LO structure functions.

Manifestly covariant representation

First we have the unpolarized structure functions:

$$F_1(x)=\frac{M}{2}\left(A+\frac{B}{\gamma}\right),\quad F_2(x)=\frac{Pq}{2M\gamma}\left(A+\frac{3B}{\gamma}\right),\tag{4}$$

where

$$A=\frac{1}{Pq}\int G\left(\frac{Pp}{M}\right)[pq-m^2]\delta\left(\frac{pq}{Pq}-x\right)\frac{\mathrm{d}^3p}{p_0}\,,\tag{5}$$
$$B=\frac{1}{Pq}\int G\left(\frac{pP}{M}\right)\left[\left(\frac{Pp}{M}\right)^2+\frac{(Pp)(Pq)}{M^2}-\frac{pq}{2}\right]\times\delta\left(\frac{pq}{Pq}-x\right)\frac{\mathrm{d}^3p}{p_0}\tag{6}$$

and

$$\gamma=1-\left(\frac{Pq}{Mq}\right)^2.\tag{7}$$

The functions $F_1=MW_1$ and $F_2=(Pq/M)W_2$ follow from the tensor equation

$$\left(-g_{\alpha\beta}+\frac{q_\alpha q_\beta}{q^2}\right)W_1+\left(P_\alpha-\frac{Pq}{q^2}q_\alpha\right)\left(P_\beta-\frac{Pq}{q^2}q_\beta\right)\frac{W_2}{M^2}$$
$$=\int G\left(\frac{pP}{M}\right)[2p_\alpha p_\beta+p_\alpha q_\beta+q_\alpha p_\beta-g_{\alpha\beta}pq]\times\delta\left((p+q)^2-m^2\right)\frac{\mathrm{d}^3p}{p_0}\,.\tag{8}$$

[1] In [12, 13] we used a different notation for the distributions defined by (2) and (3): $G_k^{\pm}$, ΔG_k and ΔG were denoted $h_{k\pm}$, Δh_k and H. Apart from that we assumed for simplicity that only three (valence) quarks contribute to the sums (2) and (3). In the present paper we assume contributions of all the quarks and antiquarks, but apparently the general form of relations (4)–(7) and (10)–(12) is in the LO approach independent of the chosen set of quarks.

The modification of the delta function term

$$\delta((p+q)^2-m^2) = \delta(2pq+q^2) = \delta\left(2Pq\left(\frac{pq}{Pq}-\frac{Q^2}{2Pq}\right)\right) = \frac{1}{2Pq}\delta\left(\frac{pq}{Pq}-x\right), \quad q^2=-Q^2, \qquad x=\frac{Q^2}{2Pq}, \tag{9}$$

introduces the dependence on the Bjorken variable x. Then contracting with the tensors $g_{\alpha\beta}$ and $P_\alpha P_\beta$ gives the set of two equations, which determine the functions F_1 and F_2 in accordance with (4)–(7).

Next, we treat the polarized structure functions. As follows from [12] the corresponding spin functions in covariant form read

$$g_1 = Pq\left(G_S-\frac{Pq}{qS}G_P\right), \quad g_2=\frac{(Pq)^2}{qS}G_P, \tag{10}$$

where S is the spin polarization vector of the nucleon, and the functions G_P and G_S are defined by

$$G_P=\frac{m}{2Pq}\int \Delta G\left(\frac{pP}{M}\right)\frac{pS}{pP+mM} \times\left[1+\frac{1}{mM}\left(pP-\frac{pu}{qu}Pq\right)\right]\delta\left(\frac{pq}{Pq}-x\right)\frac{\mathrm{d}^3p}{p_0}, \tag{11}$$

$$G_S=\frac{m}{2Pq}\int \Delta G\left(\frac{pP}{M}\right)\left[1+\frac{pS}{pP+mM}\frac{M}{m}\left(pS-\frac{pu}{qu}qS\right)\right] \times\delta\left(\frac{pq}{Pq}-x\right)\frac{\mathrm{d}^3p}{p_0}, \tag{12}$$

$$u=q+(qS)S-\frac{(Pq)}{M^2}P.$$

Rest frame representation

We now come to the rest frame representation for $Q^2 \gg 4M^2x^2$. As follows from the appendix in [12], if $Q^2 \gg 4M^2x^2$ and the above integrals are expressed in terms of the rest frame variables of the nucleon, then one can substitute

$$\frac{pq}{Pq}\rightarrow\frac{p_0+p_1}{M},$$

and the structure functions are simplified as follows:

$$F_1(x)=\frac{Mx}{2}\int G(p_0)\delta\left(\frac{p_0+p_1}{M}-x\right)\frac{\mathrm{d}^3p}{p_0}, \tag{13}$$

$$F_2(x)=Mx^2\int G(p_0)\delta\left(\frac{p_0+p_1}{M}-x\right)\frac{\mathrm{d}^3p}{p_0}, \tag{14}$$

$$g_1(x)=\frac{1}{2}\int \Delta G(p_0)\left(m+p_1+\frac{p_1^2}{p_0+m}\right) \times\delta\left(\frac{p_0+p_1}{M}-x\right)\frac{\mathrm{d}^3p}{p_0}, \tag{15}$$

$$g_2(x)=-\frac{1}{2}\int \Delta G(p_0)\left(p_1+\frac{p_1^2-p_{\mathrm{T}}^2/2}{p_0+m}\right) \times\delta\left(\frac{p_0+p_1}{M}-x\right)\frac{\mathrm{d}^3p}{p_0}, \tag{16}$$

where p_1 and p_{T} are the longitudinal and transversal momentum components of the quark. These structure functions consist of terms like

$$q(x)=Mx\int G_q(p_0)\delta\left(\frac{p_0+p_1}{M}-x\right)\frac{\mathrm{d}^3p}{p_0}, \tag{17}$$

$$\Delta q(x)=\int \Delta G_q(p_0)\left(m+p_1+\frac{p_1^2}{p_0+m}\right) \times\delta\left(\frac{p_0+p_1}{M}-x\right)\frac{\mathrm{d}^3p}{p_0}, \tag{18}$$

which correspond to the contributions from different quark flavors, $q=u,\bar{u},d,\bar{d},s,\bar{s},\ldots$ Let us remark, in the limit of the IMF approach (see next paragraph), that these functions represent probabilistic distributions of the quark momentum in terms of the fraction x of the momentum of the nucleon, $p=xP$. However, the content and interpretation of the functions (17) and (18) depending on the Bjorken variable x is more complex; their form reflects in a nontrivial way the intrinsic 3D motion of quarks.

Standard IMF representation

On the standard IMF representation we remark the following. The usual formulation of the QPM gives the known relations between the structure functions and the parton distribution functions [10]:

$$F_1(x)=\frac{1}{2}\sum_q e_q^2 q(x), \qquad F_2(x)=x\sum_q e_q^2 q(x), \tag{19}$$

$$g_1(x)=\frac{1}{2}\sum_q e_q^2\Delta q(x), \quad g_2(x)=0, \tag{20}$$

where the functions

$$q(x)=q^+(x)+q^-(x), \quad \Delta q(x)=q^+(x)-q^-(x) \tag{21}$$

represent probabilistic distributions of the momentum fraction x of the quark in the IMF. In Appendix A we have proved that these relations represent the particular, limit case of the covariant relations (4) and (10).

The three versions of the relations between the structure functions and the quark distributions can be compared. If we skip the function g_2 in the IMF representation, then the relations (19) and (20) practically represent the identity between the structure functions and distributions of the quark momentum fraction. Such simple relations are valid only for the IMF approach based on the approximation (A.1), which means that the intrinsic motion of the quarks is suppressed. In the more general versions (the covariant and the rest frame representation), where the intrinsic motion is allowed, the relations are more complex. The intrinsic motion strongly modifies also the g_2. In the standard IMF representation one has $g_2(x)=0$, but $g_2(x)\neq 0$ in the covariant and the rest frame representations.

The rest frame representation allows one to easily calculate the dependence of the first moment Γ_1 on the rate of intrinsic motion. A more detailed discussion follows in the next section. The same approach implies that the functions g_1 and g_2 for massless quarks satisfy a relation equivalent to the Wanzura–Wilczek term and obey some well known sum rules, as is shown in [12].

The functions F_1 and F_2 exactly satisfy the Callan–Gross relation $F_2(x)/F_1(x)=2x$ in the rest frame and the IMF representations, but this relation is satisfied only approximately in the manifestly covariant representation: $F_2(x)/F_1(x)\approx 2x+O(4M^2x^2/Q^2)$.

The task which was solved in the different approximations above can be formulated as follows: how can one obtain the structure functions F_1, F_2 and g_1, g_2 from the probabilistic distributions G and ΔG defined by (2) and (3)? But now we will study the inverse problem; the aim is to find a rule for obtaining the distribution functions G and ΔG from the structure functions. In the present paper we consider the functions F_2 and g_1 represented by (14) and (15). As follows from Appendix A in [13], the function

$$V_n(x)=\int K(p_0)\left(\frac{p_0}{M}\right)^n\delta\left(\frac{p_0+p_1}{M}-x\right)\mathrm{d}^3p \qquad (22)$$

satisfies

$$V_n'(x_\pm)x_\pm=\mp 2\pi MK(\xi)\xi\sqrt{\xi^2-m^2}\left(\frac{\xi}{M}\right)^n;$$
$$x_\pm=\frac{\xi\pm\sqrt{\xi^2-m^2}}{M}. \qquad (23)$$

In this section we consider only the case $m\to 0$; then we have

$$V_n'(x)x=-2\pi MK(\xi)\xi^2\left(\frac{\xi}{M}\right)^n;\quad x=\frac{2\xi}{M}. \qquad (24)$$

As we shall see below, with the use of this relation one can obtain the probabilistic distributions $G(p)$ and $\Delta G(p)$ from the experimentally measured structure functions. The same procedure will be applied to get $G_q(p)$ and $\Delta G_q(p)$ from the usual parton distributions $q(x)$ and $\Delta q(x)$, defined by (19) and (20).

Let us remark that in the present stage QCD evolution is not included into the model. However, this fact does not represent any restriction for the present purpose: to obtain information about the distributions of the quarks at some scale Q^2 from the structure functions measured at the same Q^2. The distribution of the gluons is another part of the nucleon picture. But since our present discussion is directed to the relation between the structure functions and the corresponding distributions of the quarks at a given scale, the gluon distribution is left aside.

2.1 Momentum distribution from structure function F_2

In accordance with the definition (22), in which the distribution $K(p_0)$ is substituted for by $G(p_0)$, the structure function (14) can be written in the form

$$F_2(x)=x^2V_{-1}(x). \qquad (25)$$

Then, with the use of (24), one gets

$$G(p)=-\frac{1}{\pi M^3}\left(\frac{F_2(x)}{x^2}\right)'$$
$$=\frac{1}{\pi M^3x^2}\left(\frac{2F_2(x)}{x}-F_2'(x)\right);$$
$$x=\frac{2p}{M},\qquad p\equiv\sqrt{\mathbf{p}^2}=p_0, \qquad (26)$$

which in terms of the quark distributions means

$$G_q(p)=-\frac{1}{\pi M^3}\left(\frac{q(x)}{x}\right)'=\frac{1}{\pi M^3x^2}(q(x)-xq'(x)). \qquad (27)$$

The probability distribution G_q measures the number of quarks of flavor q in the element d^3p. Since $\mathrm{d}^3p=4\pi p^2\,\mathrm{d}p$, the distribution measuring the number of quarks in the element $\mathrm{d}p/M$ reads

$$P_q(p)=4\pi p^2MG_q(p)=-x^2\left(\frac{q(x)}{x}\right)'=q(x)-xq'(x). \qquad (28)$$

The probability distribution $G_q(p)$ is positive, so the last relation implies

$$\left(\frac{q(x)}{x}\right)'\leq 0. \qquad (29)$$

Let us note that the maximum value of the momentum of the quark is $p_{\max}=M/2$, which is a consequence of the kinematics in the nucleon rest frame, where the single quark momentum must be compensated by the momentum of the other partons.

Another quantity that can be obtained is the distribution of the transversal momentum of the quarks. Obviously the integral

$$\frac{\mathrm{d}N_q}{\mathrm{d}p_\mathrm{T}^2}=\int G_q(p)\delta\left(p_2^2+p_3^2-p_\mathrm{T}^2\right)\,\mathrm{d}^3p, \qquad (30)$$

which represents the number of quarks in the element $\mathrm{d}p_\mathrm{T}^2$, can be modified as

$$\frac{\mathrm{d}N_q}{\mathrm{d}p_\mathrm{T}^2}=2\pi\int_0^{\sqrt{p_{\max}^2-p_\mathrm{T}^2}}G_q\left(\sqrt{p_1^2+p_\mathrm{T}^2}\right)\mathrm{d}p_1. \qquad (31)$$

It follows that the distribution corresponding to the number of quarks in the element $\mathrm{d}p_\mathrm{T}/M$ reads

$$P_q(p_\mathrm{T})=M\frac{\mathrm{d}N_q}{\mathrm{d}p_\mathrm{T}}$$
$$=4\pi p_\mathrm{T}M\int_0^{\sqrt{p_{\max}^2-p_\mathrm{T}^2}}G_q\left(\sqrt{p_1^2+p_\mathrm{T}^2}\right)\mathrm{d}p_1. \qquad (32)$$

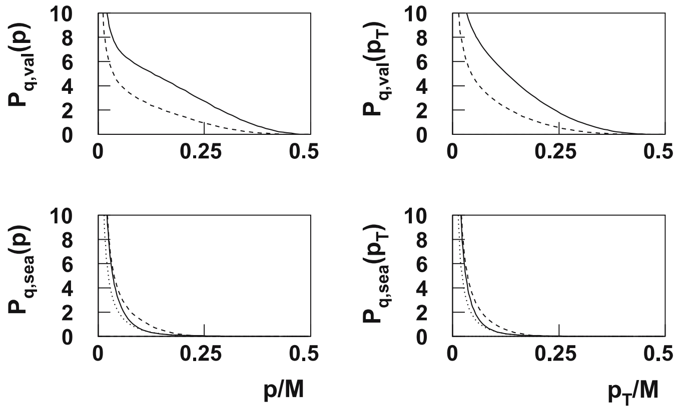

Fig. 1. The quark momentum distributions in the rest frame of the proton: the p and p_{T} distributions for valence quarks $P_{q,\mathrm{val}} = P_q - P_{\bar{q}}$ and sea quarks $P_{\bar{q}}$ at $Q^2 = 4\,\mathrm{GeV}^2$. Notation: $u, \bar{u}$ is indicated by a *solid line*, $d, \bar{d}$ by a *dashed line* and $\bar{s}$ by a *dotted line*

Then, with the use of (28), one gets the distribution

$$P_q(p_{\mathrm{T}}) = \frac{4p_{\mathrm{T}}}{M^2} \int_0^{\sqrt{p_{\max}^2 - p_{\mathrm{T}}^2}} \frac{1}{x^2}(q(x) - xq'(x))\,\mathrm{d}p_1\,,$$
$$x = \frac{2\sqrt{p_1^2 + p_{\mathrm{T}}^2}}{M}\,. \tag{33}$$

In Fig. 1 the distributions (28) and (33) are displayed for the valence and sea quarks. As input we used the standard parameterization [28] of the parton distribution functions $q(x)$ and $\bar{q}(x)$ (LO at the scale $4\,\mathrm{GeV}^2$). The resulting distributions P_q and $P_{\bar{q}}$ are positive, and this means that the input distributions q and $\bar{q}$ satisfy the constraint (29).

Using (27) one can calculate the mean values

$$\langle p \rangle_q = \frac{\int p G_q(p)\,\mathrm{d}^3 p}{\int G_q(p)\,\mathrm{d}^3 p} = \frac{M}{2} \frac{\int_0^1 x(q(x) - xq'(x))\,\mathrm{d}x}{\int_0^1 (q(x) - xq'(x))\,\mathrm{d}x}\,. \tag{34}$$

In the case of sea quarks extrapolation of the distribution functions for $x \to 0$ gives a divergent integral in the denominator, and it follows that $\langle p \rangle_{\mathrm{sea}} \to 0$. For the valence quarks $q_{\mathrm{val}} = q - \bar{q}$ this integral converges and integration by parts gives

$$\langle p \rangle_{q,\mathrm{val}} = \frac{3M}{4} \frac{\int_0^1 x q_{\mathrm{val}}(x)\,\mathrm{d}x}{\int_0^1 q_{\mathrm{val}}(x)\,\mathrm{d}x}\,. \tag{35}$$

Calculation of $\langle p \rangle_{q,\mathrm{val}}$ gives roughly $0.11\,\mathrm{GeV}/c$ for u and $0.083\,\mathrm{GeV}/c$ for d quarks. Since $G_q(p)$ has rotational symmetry, the average transversal momentum can be calculated to be $\langle p_{\mathrm{T}} \rangle = \pi/4 \cdot \langle p \rangle$.

2.2 Momentum distribution from structure function g_1

In (44) of [13] we proved that

$$g_1(x) = V_0(x) - \int_x^1 \left(4\frac{x^2}{y^3} - \frac{x}{y^2}\right) V_0(y)\,\mathrm{d}y\,, \tag{36}$$

where the function V_0 is defined by (22) for $n = 0$ and $K(p) = \Delta G(p)$. In Appendix B it is shown that the last relation can be modified to:

$$V_{-1}(x) = \frac{2}{x}\left(g_1(x) + 2\int_x^1 \frac{g_1(y)}{y}\,\mathrm{d}y\right). \tag{37}$$

Then, in an accordance with (24), we obtain

$$V'_{-1}(x) = -\pi M^3 \Delta G(p)\,, \quad x = \frac{2p}{M}\,, \tag{38}$$

$$\Delta G(p) = -\frac{2}{\pi M^3}\left[\frac{1}{x}\left(g_1(x) + 2\int_x^1 \frac{g_1(y)}{y}\,\mathrm{d}y\right)\right]' \tag{39}$$

or

$$\Delta G(p) = \frac{2}{\pi M^3 x^2}\left(3g_1(x) + 2\int_x^1 \frac{g_1(y)}{y}\,\mathrm{d}y - xg_1'(x)\right). \tag{40}$$

Now we substitute

$$\Delta q(x) = 2g_1(x)\,,$$
$$g_1(x) + g_2(x) = \int_x^1 \frac{g_1(y)}{y}\,\mathrm{d}y = \Delta q_{\mathrm{T}}(x)/2 \tag{41}$$

and next we shall consider the flavors separately. The second equality represents the Wanzura–Wilczek relation for the twist-2 approximation of g_2, which is valid for the present approach, as proved in [13]. Now (40) in terms of the quark distributions reads

$$\Delta G_q(p) = \frac{1}{\pi M^3 x^2}\left(3\Delta q(x) + 2\int_x^1 \frac{\Delta q(y)}{y}\,\mathrm{d}y - x\Delta q'(x)\right), \tag{42}$$

or, equivalently, with the use of (39) and (41),

$$\Delta G_q(p) = -\frac{1}{\pi M^3}\left(\frac{\Delta q(x) + 2\Delta q_{\mathrm{T}}(x)}{x}\right)'; \quad x = \frac{2p}{M}\,. \tag{43}$$

Obviously the distribution ΔG_q together with the distribution (27) allows one to obtain the polarized distributions $G_q^{\pm}$ as follows:

$$G_q^{\pm}(p) = \frac{1}{2}(G_q(p) \pm \Delta G_q(p))\,. \tag{44}$$

The distributions ΔG_q and $G_q^{\pm}$ measure the number of quarks in the element $\mathrm{d}^3 p$. They can be replaced, similarly as the distribution G_q in (28), by the distributions ΔP_q and $P_q^{\pm}$, measuring the number of quarks in the element $\mathrm{d}p/M$:

$$\Delta P_q(p) = 3\Delta q(x) + 2\int_x^1 \frac{\Delta q(y)}{y}\,\mathrm{d}y - x\Delta q'(x)\,, \tag{45}$$

$$P_q^{\pm}(p) = \frac{1}{2}(q(x) - xq'(x))$$
$$\pm\left(\frac{3}{2}\Delta q(x) + \int_x^1 \frac{\Delta q(y)}{y}\,\mathrm{d}y - \frac{x}{2}\Delta q'(x)\right). \tag{46}$$

Obviously the probability distributions should satisfy

$$|\Delta G_q(p)| \leq G_q(p)\,, \tag{47}$$

which after inserting the results from (43) and (27) implies

$$\left|\left(\frac{\Delta q(x)+2\Delta q_{\mathrm{T}}(x)}{x}\right)'\right| \leq -\left(\frac{q(x)}{x}\right)', \tag{48}$$

where positivity of the right hand side was required in (29). Another self-consistency test of the approach is represented by the inequality

$$|\Delta q(x)| \leq q(x)\,, \tag{49}$$

which is proved in Appendix C.

With the use of (17) one can formally calculate the partial structure functions corresponding to the subsets of positively and negatively polarized quarks:

$$f_q^{\pm}(x) = Mx\int G_q^{\pm}(p)\delta\left(\frac{p_0+p_1}{M}-x\right)\frac{\mathrm{d}^3p}{p_0}\,. \tag{50}$$

Apparently the following equation holds:

$$f_q(x) \equiv f_q^+(x)+f_q^-(x) = q(x)\,, \tag{51}$$

and one can also define

$$\Delta f_q(x) = f_q^+(x)-f_q^-(x)\,, \tag{52}$$

or, equivalently,

$$\Delta f_q(x) = Mx\int \Delta G_q(p)\delta\left(\frac{p_0+p_1}{M}-x\right)\frac{\mathrm{d}^3p}{p_0}\,. \tag{53}$$

Obviously we have

$$f_q^{\pm}(x) = \frac{1}{2}(f_q(x)\pm\Delta f_q(x))\,, \tag{54}$$

and (47) implies

$$|\Delta f_q(x)| \leq q(x)\,. \tag{55}$$

Let us note that $f_q^+ + f_q^- = q$, but $f_q^+ - f_q^- \neq \Delta q$ in the sense of (17) and (18). The last inequality is replaced by equality only in the limit of the IMF approach. The relation (53) can be written

$$\Delta f_q(x) = xV_{q,-1}(x)\,, \tag{56}$$

where

$$V_{q,-1}(x) = M\int \Delta G_q(p)\delta\left(\frac{p_0+p_1}{M}-x\right)\frac{\mathrm{d}^3p}{p_0}\,. \tag{57}$$

At the same time (37) can be replaced by

$$V_{q,-1}(x) = \frac{1}{x}\left(\Delta q(x)+2\int_x^1\frac{\Delta q(y)}{y}\,\mathrm{d}y\right), \tag{58}$$

which, after inserting the result from (56), gives

$$\Delta f_q(x) = \Delta q(x)+2\int_x^1\frac{\Delta q(y)}{y}\,\mathrm{d}y. \tag{59}$$

This equality together with (55) gives

$$\left|\Delta q(x)+2\int_x^1\frac{\Delta q(y)}{y}\,\mathrm{d}y\right| \leq q(x)\,, \tag{60}$$

or, equivalently,

$$|\Delta q(x)+2\Delta q_{\mathrm{T}}(x)| \leq q(x)\,. \tag{61}$$

Now, using the input on $q(x)$ [28] and $\Delta q(x)$ [29] (LO at the scale 4 GeV^2) one can calculate the distributions ΔP_q, P_q and $P_q^{\pm}$ and the related structure functions Δf_q, f_q and $f_q^{\pm}$. The result is displayed in Fig. 2 and one can observe the following.

Positivity of the distributions $P_q^{\pm}$ and $f_q^{\pm}$ implies that the self-consistency tests (47) and (55) and their equivalents (48) and (60) are satisfied with the exception of a small negative disturbance in $G_u^-(P_u^-)$ and f_u^-. A possible reason is that the results of the two different procedures for fitting $q(x)$ and $\Delta q(x)$ are combined and some uncertainty is unavoidable.

The mean value of the distribution ΔG_q can be estimated to be

$$\langle p\rangle_q = \frac{\int p\Delta G_q(p)\,\mathrm{d}^3p}{\int \Delta G_q(p)\,\mathrm{d}^3p} = \frac{M}{2}\frac{\int_0^1 x\Delta q(x)\,\mathrm{d}x}{\int_0^1 \Delta q(x)\,\mathrm{d}x}\,. \tag{62}$$

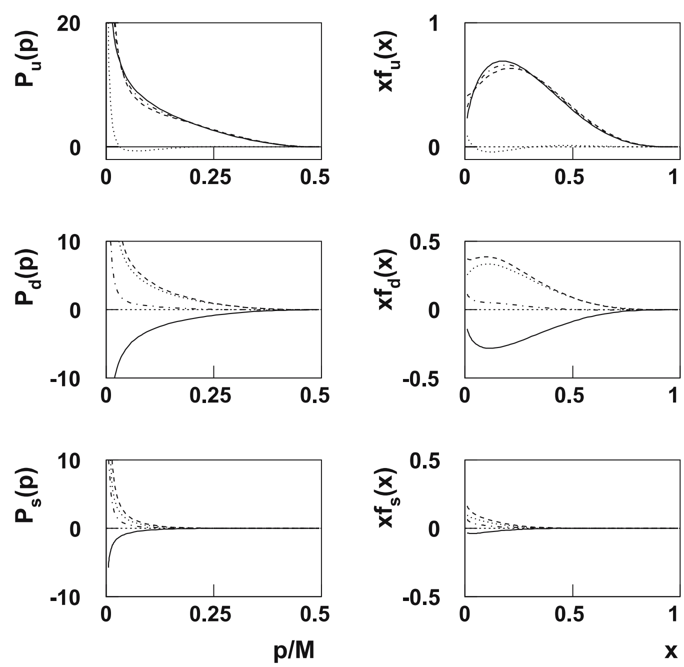

Fig. 2. The probability distributions $\Delta P_q, P_q, P_q^+$ and P_q^- of the u, d, s quarks, and the related structure functions Δf_q, f_q, f_q^+ and f_q^- are represented by the *solid*, *dashed*, *dash-and-dot* and *dotted lines*

The proof of this relation is given in Appendix D. The numerical calculation gives 0.090 GeV/c for the u and 0.070 GeV/c for the d quarks. These numbers are well comparable with those calculated from (35), which correspond to the valence quarks. Also the shape of the distributions $x\Delta f_u(x)$ and $x\Delta f_d(x)$ is very similar to that of the valence terms. In other words, the results confirm that the spin contribution of the quarks comes dominantly from the valence region.

Due to the input values with $\Delta u(x) > 0$ and $\Delta d(x) < 0$ one can expect that $P_u^+ \geq P_u^-$, $P_d^- \geq P_d^+$, $f_u^+ \geq f_u^-$ and $f_d^- \geq f_d^+$. Besides, the curves in the figure show that P_u^-, P_d^+, f_u^- and f_d^+ are close to zero, at least in the valence region.

3 Intrinsic quark motion and orbital momentum

The rules of quantum mechanics say that angular momentum consists of an orbital and a spin part, $\mathbf{j} = \mathbf{l} + \mathbf{s}$, and that in the relativistic case the quantities $\mathbf{l}$ and $\mathbf{s}$ are not conserved separately, but only the total angular momentum $\mathbf{j}$ is conserved. This simple fact was in the context of quarks inside the nucleon pointed out in [30]. It means that only j^2 and j_z are well-defined quantum numbers and the corresponding states of the particle with spin 1/2 are represented by the bispinor spherical waves [31]

$$\psi_{kjlj_z}(\mathbf{p}) = \frac{\delta(p-k)}{p\sqrt{2p_0}} \begin{pmatrix} \mathrm{i}^{-l}\sqrt{p_0+m}\,\Omega_{jlj_z}(\omega) \\ \mathrm{i}^{-\lambda}\sqrt{p_0-m}\,\Omega_{j\lambda j_z}(\omega) \end{pmatrix}, \tag{63}$$

where $\omega = \mathbf{p}/p$, $l = j \pm \frac{1}{2}$, $\lambda = 2j - l$ (l defines the parity) and

$$\Omega_{j,l,j_z}(\omega) = \begin{pmatrix} \sqrt{\frac{j+j_z}{2j}}\,Y_{l,j_z-1/2}(\omega) \\ \sqrt{\frac{j-j_z}{2j}}\,Y_{l,j_z+1/2}(\omega) \end{pmatrix}; \qquad l = j - \frac{1}{2},$$

$$\Omega_{j,l,j_z}(\omega) = \begin{pmatrix} -\sqrt{\frac{j-j_z+1}{2j+2}}\,Y_{l,j_z-1/2}(\omega) \\ \sqrt{\frac{j+j_z+1}{2j+2}}\,Y_{l,j_z+1/2}(\omega) \end{pmatrix}; \quad l = j + \frac{1}{2}.$$

The states are normalized by

$$\int \psi^\dagger_{k'j'l'j'_z}(\mathbf{p})\psi_{kjlj_z}(\mathbf{p})\,\mathrm{d}^3p = \delta(k-k')\delta_{jj'}\delta_{ll'}\delta_{j_zj'_z}. \tag{64}$$

The wavefunction (63) is simplified for $j = j_z = 1/2$ and $l = 0$. Taking into account that

$$Y_{00} = \frac{1}{\sqrt{4\pi}}, \qquad Y_{10} = \mathrm{i}\sqrt{\frac{3}{4\pi}}\cos\theta,$$

$$Y_{11} = -\mathrm{i}\sqrt{\frac{3}{8\pi}}\sin\theta\exp(\mathrm{i}\varphi),$$

one gets

$$\psi_{kjlj_z}(\mathbf{p}) = \frac{\delta(p-k)}{p\sqrt{8\pi p_0}} \begin{pmatrix} \sqrt{p_0+m}\begin{pmatrix}1\\0\end{pmatrix} \\ -\sqrt{p_0-m}\begin{pmatrix}\cos\theta \\ \sin\theta\exp(\mathrm{i}\varphi)\end{pmatrix} \end{pmatrix}. \tag{65}$$

Let us note that $j = 1/2$ is the minimum angular momentum for a particle with spin 1/2. If one considers the quark state as a superposition,

$$\Psi(\mathbf{p}) = \int a_k\psi_{kjlj_z}(\mathbf{p})\,\mathrm{d}k, \qquad \int a_k^\star a_k\,\mathrm{d}k = 1, \tag{66}$$

then its average spin contribution to the total angular momentum reads

$$\langle s\rangle = \int \Psi^\dagger(\mathbf{p})\Sigma_z\Psi(\mathbf{p})\,\mathrm{d}^3p, \quad \Sigma_z = \frac{1}{2}\begin{pmatrix}\sigma_z & \cdot \\ \cdot & \sigma_z\end{pmatrix}. \tag{67}$$

After inserting (65) and (66) into (67) one gets

$$\begin{aligned}\langle s\rangle &= \int a_p^\star a_p \frac{(p_0+m)+(p_0-m)(\cos^2\theta-\sin^2\theta)}{16\pi p^2 p_0}\,\mathrm{d}^3p \\ &= \frac{1}{2}\int a_p^\star a_p\left(\frac{1}{3}+\frac{2m}{3p_0}\right)\mathrm{d}p.\end{aligned} \tag{68}$$

Since $j = 1/2$, the last relation implies for the orbital momentum of the quark that

$$\langle l\rangle = \frac{1}{3}\int a_p^\star a_p\left(1-\frac{m}{p_0}\right)\mathrm{d}p. \tag{69}$$

This means that for quarks in the state $j = j_z = 1/2$ there are the following extreme scenarios.

Either one has massive and static quarks ($p_0 = m$), which implies that $\langle s\rangle = j = 1/2$ and $\langle l\rangle = 0$. This is evident, since without kinetic energy no orbital momentum can be generated.

But another possibility is that one has massless quarks ($m \ll p_0$), which implies that $\langle s\rangle = 1/6$ and $\langle l\rangle = 1/3$.

Generally, for $p_0 \geq m$, one gets $1/3 \leq \langle s\rangle/j \leq 1$. In other words, for the states with $p_0 > m$, part of the total angular momentum $j = 1/2$ is necessarily generated by the orbital momentum. This is a consequence of quantum mechanics, and not a consequence of the particular model. If one assumes the effective mass of the quark to be of the order of thousandths and the intrinsic momentum to be of the order of tenths of GeV, which is a quite realistic assumption, then the second scenario is clearly preferred. Further, the mean kinetic energy corresponding to the superposition (66) reads

$$\langle E_{\mathrm{kin}}\rangle = \int a_p^\star a_p E_{\mathrm{kin}}\,\mathrm{d}p; \quad E_{\mathrm{kin}} = p_0 - m, \tag{70}$$

and at the same time (69) can be rewritten

$$\langle l\rangle = \frac{1}{3}\int a_p^\star a_p \frac{E_{\mathrm{kin}}}{p_0}\,\mathrm{d}p. \tag{71}$$

It is evident that for fixed $j = 1/2$ both quantities are almost equivalent in the nucleon rest frame: more kinetic energy generates more orbital momentum and vice versa.

Further, the average spin part $\langle s \rangle$ of the total angular momentum $j = 1/2$ related to a single quark according to (68) can be compared to the integral

$$\Gamma_1 = \int_0^1 g_1(x)\,\mathrm{d}x\,, \tag{72}$$

which measures the total quark spin contribution to the spin of the nucleon. For g_1 in (15) this integral reads

$$\Gamma_1 = \frac{1}{2}\int \Delta G(p_0)\left(\frac{1}{3} + \frac{2m}{3p_0}\right)\mathrm{d}^3p\,. \tag{73}$$

The dependence of the integrals (68) and (73) on the intrinsic motion is controlled by the same term $(1/3 + 2m/3p_0)$, which in both cases has its origin in the covariant kinematics of the particle with $s = 1/2$. In fact, the procedures for the calculation of these integrals are based on the two different representations of the solutions of the Dirac equation: plane waves (1) and spherical waves (65). It is apparent that for the scenario of massless quarks ($m \ll p_0$), due to the necessary presence of the orbital motion, both numbers Γ_1 and $\langle s \rangle$ are roughly three times less than for the scenario of massive and static quarks ($m \simeq p_0$). What is the underlying physics behind the interplay between the spin and orbital momentum? Actually, speaking about the spin of the particle represented by the state (1), one should take into account the following.

The definite projection of the spin in the direction $\mathbf{n}$ is a well-defined quantum number only for the particle at rest ($p = 0$) or for the particle moving in the direction $\mathbf{n}$, i.e. $\mathbf{p}/p = \pm\mathbf{n}$. In these cases we have

$$s = u^{\dagger}(\mathbf{p}, \lambda\mathbf{n})\mathbf{n}\boldsymbol{\Sigma}u(\mathbf{p}, \lambda\mathbf{n}) = \pm 1/2\,. \tag{74}$$

But in other cases, as shown in Appendix E, only the inequality

$$\langle s \rangle = |u^{\dagger}(\mathbf{p}, \lambda\mathbf{n})\mathbf{n}\boldsymbol{\Sigma}u(\mathbf{p}, \lambda\mathbf{n})| < 1/2 \tag{75}$$

is satisfied. Roughly speaking, the result of measuring the spin of a quark depends on its momentum in the given reference frame (the rest frame of the nucleon). This obvious effect acts also in the states that are represented by the superposition of plane waves (1) with different momenta $\mathbf{p}$ and resulting in $\langle \mathbf{p} \rangle = 0$, but with $\langle \mathbf{p}^2 \rangle > 0$. In [12] we showed that averaging of the spin projection (75) over the spherical momentum distribution gives the result equivalent to (73). The state (66) can also be decomposed into plane waves having a spherical momentum distribution and the spin mean value given by (68). The well-defined quantum numbers $j = j_z = 1/2$ imply that the spin reduction due to an increasing intrinsic kinetic energy is compensated by an increasing orbital momentum.

Now, what does the preferred scenario of massless quarks ($\langle m/p_0 \rangle \ll 1$) imply for the spin structure of the whole nucleon, and what are the integral quark spin and orbital contributions to the spin of the nucleon? Obviously, using some input on the total quark longitudinal polarization $\Delta\Sigma$, one can estimate the relative quark spin and orbital contributions to be

$$S = \Delta\Sigma\,, \quad L = 2\Delta\Sigma\,, \quad \Delta\Sigma = \sum_q \int_0^1 \Delta q(x)\,\mathrm{d}x\,. \tag{76}$$

Table 1. Relative integral contributions of the quark spins (S), orbital momenta (L) and their sum (J) to the total spin of the nucleon. Results are shown of our calculation (*right*) and the prediction of the CQSM model (*left*)

	CQSM		Present paper	
	$Q^2 = 0.3\,\mathrm{GeV}^2$	$Q^2 = 4\,\mathrm{GeV}^2$	$\Delta\Sigma = 0.2$	$\Delta\Sigma = 0.3$
S [%]	35.0	31.8	20.0	30.0
L [%]	65.0	35.8	40.0	60.0
J [%]	100.0	67.6	60.0	90.0

At the same time our approach can be compared with the calculation based on the chiral quark soliton model (CQSM) [24, 25], in which a significant role for the orbital momentum of the quark is assumed as well. In Table 1 some results of both models are shown. In spite of some similarity between the two sets of numbers, there are substantial differences between both approaches. Let us mention at least the two that seem to be most evident.

First, the presence of a significant fraction of the orbital momentum in the CQSM apparently follows from the dynamics inherent in the model. On the other hand, in our approach the important role of the orbital momentum follows from the kinematics, so it should not be too sensitive to the details of the inherent dynamics. Actually the effect takes place in LO when quarks interacting with the probing photon can be effectively described as free fermions in states like (66) with a sufficiently low effective ratio $\langle m/p_0 \rangle$, which controls the fraction of orbital momentum (69). Of course, the value of this ratio itself is a question of the dynamics.

Second, in the CQSM antiquarks are predicted to have opposite signs for the spin and orbital contributions. In our approach the two contributions are proportional and have the same signs regardless of flavor or antiflavor.

A last comment concerns the total angular momentum of the quarks, J, by which room for the gluon contribution J_g is defined. Results in Table 1 related to the CQSM suggest that a higher Q^2 implies a greater gluon contribution. Our results suggest that the gluon contribution can be rather sensitive to the longitudinal polarization: for $\Delta\Sigma \simeq 1/3$, 0.3 and 0.2 the gluon contribution can represent $\simeq 0$, 10 and 40%, respectively. So the value empirically known [25],

$$\Delta\Sigma \simeq 0.2\text{–}0.35\,, \tag{77}$$

does not exclude any of these possibilities.

4 Summary and conclusion

We studied a covariant version of the QPM with spherically symmetric distributions of the quark momentum in the rest frame of the nucleon. The main results obtained in this paper can be summarized as follows.

The relations between the distribution functions $q(x)$, $\Delta q(x)$ and the corresponding 3D momentum distributions $G_q^{\pm}(p) = G_q(p) \pm \Delta G_q(p)$ of the quarks are obtained. In this way the momentum distributions of the positively and negatively polarized quarks $G_q^{\pm}(p)$ are calculated from the experimentally measured structure functions F_2 and g_1. At the same time these relations, due to positivity of the probabilistic distributions G_q and $G_q^{\pm}$, imply some inequalities for $q(x)$ and $\Delta q(x)$. We proved that these constraints, serving as self-consistency tests of the approach, are satisfied.

Next, we showed that an important role of the orbital momentum of the quark emerges as a direct consequence of a covariant description. Since in the relativistic case only the total angular momentum $\mathbf{j} = \mathbf{l} + \mathbf{s}$ is a well-defined quantum number, there arises some interplay between its spin and orbital parts. For the quark in the state, $j_z = 1/2$, as a result of this interplay its spin part is reduced in favor of the orbital one. The role of the orbital motion increases with the rate of the intrinsic motion of the quark; for $\langle m/p_0 \rangle \ll 1$ its fraction reaches $\langle l_z \rangle = 2/6$, whereas $\langle s_z \rangle = 1/6$ only. Simultaneously this effect is truly reproduced also in the formalism of structure functions, and in this connection some implications for the global spin structure of the nucleon were suggested.

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Appendix A: Structure functions in the approach of the infinite momentum frame

The necessary condition for obtaining the equalities (19) and (20) is the covariant relation

$$p_\alpha = y P_\alpha \,, \tag{A.1}$$

which implies

$$m = yM \,, \tag{A.2}$$

and $\mathbf{p} = 0$ in the rest frame of the nucleon and $p_{\mathrm{T}} = 0$ in the IMF.

For the calculation of the integrals (5) and (6) in the IMF approach one can substitute p by yP, and d^3p/p_0 by $\pi \,\mathrm{d}p_{\mathrm{T}}^2 \,\mathrm{d}y/y$. Then, after some algebra the structure functions (4) read

$$F_1(x) = \frac{1}{2} Mx \int G(yM)\delta(y-x)\pi \,\mathrm{d}p_{\mathrm{T}}^2 \frac{\mathrm{d}y}{y} \,,$$
$$F_2(x) = Mx^2 \int G(yM)\delta(y-x)\pi \,\mathrm{d}p_{\mathrm{T}}^2 \frac{\mathrm{d}y}{y} \,. \tag{A.3}$$

Since the approximation (A.1) implies a sharply peaked distribution at $p_{\mathrm{T}}^2 \to 0$, one can identify

$$MG_q(yM)\pi \,\mathrm{d}p_{\mathrm{T}}^2 = q(y) \,, \tag{A.4}$$

and then (19) and (A.3) after integrating are equivalent.

In the same way the equalities (10)–(12) can be modified. Taking into account that $pS \to yPS = 0$, one obtains

$$g_1(x) = \frac{m}{2} \int \Delta G(yM)\delta(y-x)\pi \,\mathrm{d}p_{\mathrm{T}}^2 \frac{\mathrm{d}y}{y} \,, \quad g_2(x) = 0 \,. \tag{A.5}$$

If we put

$$M\Delta G_q(yM)\pi \,\mathrm{d}p_{\mathrm{T}}^2 = \Delta q(y) \tag{A.6}$$

and take into account (A.2), then it is obvious that (20) and (A.5) are equivalent.

Appendix B: Proof of (37)

In [13] we proved the relation

$$\frac{V_j'(x)}{V_k'(x)} = \left(\frac{x}{2} + \frac{x_0^2}{2x} \right)^{j-k} ; \quad x_0 = \frac{m}{M} \,, \tag{B.1}$$

which for $m \to 0$ implies

$$V_0(x) = \frac{1}{2} \left(x V_{-1}(x) + \int_0^x V_{-1}(y) \,\mathrm{d}y \right) . \tag{B.2}$$

After inserting V_0 from this relation into (36) one gets

$$\begin{aligned} g_1(x) = {} & \frac{1}{2} \left(x V_{-1}(x) + \int_0^x V_{-1}(y) \,\mathrm{d}y \right) \\ & - 2x^2 \left(\int_x^1 \frac{V_{-1}(y)}{y^2} \,\mathrm{d}y + \int_x^1 \frac{1}{y^3} \int_y^1 V_{-1}(z) \,\mathrm{d}z \,\mathrm{d}y \right) \\ & + \frac{1}{2} x \left(\int_x^1 \frac{V_{-1}(y)}{y} \,\mathrm{d}y + \int_x^1 \frac{1}{y^2} \int_y^1 V_{-1}(z) \,\mathrm{d}z \,\mathrm{d}y \right) . \end{aligned} \tag{B.3}$$

The double integrals can be reduced by integration by parts with the use of

$$\int_x^1 a(y) \left(\int_y^1 b(z) \,\mathrm{d}z \right) \mathrm{d}y = \int_x^1 (A(y) - A(x)) b(y) \,\mathrm{d}y \,,$$
$$A'(x) = a(x) \,, \tag{B.4}$$

and then (B.3) is simplified:

$$g_1(x) = \frac{1}{2}xV_{-1}(x) - x^2 \int_x^1 \frac{V_{-1}(y)}{y^2}\,\mathrm{d}y\,. \tag{B.5}$$

In the next step we extract V_{-1} from this relation. After the substitution $V(x) = V_{-1}(x)/x$, the relation reads

$$\frac{g_1(x)}{x^2} = \frac{1}{2}V(x) - \int_x^1 \frac{V(y)}{y}\,\mathrm{d}y\,, \tag{B.6}$$

which implies the differential equation for $V(x)$:

$$\frac{1}{2}V'(x) + \frac{V(x)}{x} = \left(\frac{g_1(x)}{x^2}\right)'. \tag{B.7}$$

The corresponding homogeneous equation

$$\frac{1}{2}V'(x) + \frac{V(x)}{x} = 0 \tag{B.8}$$

gives the solution

$$V(x) = \frac{C}{x^2}\,, \tag{B.9}$$

which after inserting into (B.7) gives

$$C'(x) = 2x^2\left(\frac{g_1(x)}{x^2}\right)'. \tag{B.10}$$

After integration one easily gets the relation inverse to (B.5):

$$V_{-1}(x) = \frac{2}{x}\left(g_1(x) + 2\int_x^1 \frac{g_1(y)}{y}\,\mathrm{d}y\right), \tag{B.11}$$

which coincides with (37).

Appendix C: Proof of (49)

The relations (17) and (18) imply that the inequality (49) is satisfied if

$$p_0 + p_1 \geq \left| m + p_1 + \frac{p_1^2}{p_0+m}\right| = \left| p_0 + p_1 - \frac{p_\mathrm{T}^2}{p_0+m}\right|. \tag{C.1}$$

There are two cases.

First, $p_0 + p_1 - p_\mathrm{T}^2/(p_0+m) \geq 0$; then instead of (C.1) one can write

$$p_0 + p_1 \geq p_0 + p_1 - \frac{p_\mathrm{T}^2}{p_0+m}\,, \tag{C.2}$$

which is always satisfied.

Second, $p_0 + p_1 - p_\mathrm{T}^2/(p_0+m) < 0$; then (C.1) is equivalent to

$$p_0 + p_1 \geq -p_0 - p_1 + \frac{p_\mathrm{T}^2}{p_0+m} \Leftrightarrow 2(p_0+p_1) \geq \frac{p_\mathrm{T}^2}{p_0+m}\,. \tag{C.3}$$

Since

$$\begin{aligned} 2p_0 \geq p_0 - p_1 &\Rightarrow 2(p_0+m) \geq p_0 - p_1 \\ &\Rightarrow 2(p_0+m)(p_0+p_1) \geq (p_0-p_1)(p_0+p_1) \\ &\Rightarrow 2(p_0+m)(p_0+p_1) \geq p_\mathrm{T}^2 \\ &\Rightarrow 2(p_0+p_1) \geq \frac{p_\mathrm{T}^2}{p_0+m}\,, \end{aligned}$$

(C.3) is always satisfied. In this way (C.1) and (49) are proved.

Appendix D: Proof of (62)

The relation (40) implies

$$\int \Delta G_q(p)\,\mathrm{d}^3p = \frac{1}{2}\int_0^1 \left(3\Delta q(x) + 2\int_x^1 \frac{\Delta q(y)}{y}\,\mathrm{d}y - x\Delta q'(x)\right)\mathrm{d}x \tag{D.1}$$

and

$$\int p\Delta G_q(p)\,\mathrm{d}^3p = \frac{M}{4}\int_0^1 \left(3x\Delta q(x) + 2x\int_x^1 \frac{\Delta q(y)}{y}\,\mathrm{d}y - x^2\Delta q'(x)\right)\mathrm{d}x\,. \tag{D.2}$$

If one denotes

$$\Gamma_1^q = \int_0^1 \Delta q(x)\,\mathrm{d}x\,, \quad \Gamma_2^q = \int_0^1 x\Delta q(x)\,\mathrm{d}x\,, \tag{D.3}$$

then integration by parts gives

$$\int_0^1\int_x^1 \frac{\Delta q(y)}{y}\,\mathrm{d}y\,\mathrm{d}x = \Gamma_1^q\,, \quad \int_0^1 x\Delta q'(x)\,\mathrm{d}x = -\Gamma_1^q \tag{D.4}$$

and

$$\int_0^1 2x\int_x^1 \frac{\Delta q(y)}{y}\,\mathrm{d}y\,\mathrm{d}x = \Gamma_2^q\,, \quad \int_0^1 x^2\Delta q'(x)\,\mathrm{d}x = -2\Gamma_2^q\,. \tag{D.5}$$

Now, one can easily express the ratio:

$$\frac{\int p\Delta G_q(p)\,\mathrm{d}^3p}{\int \Delta G_q(p)\,\mathrm{d}^3p} = \frac{M}{2}\frac{\Gamma_2^q}{\Gamma_1^q}\,, \tag{D.6}$$

and in this way (62) is proved.

Appendix E: Proof of (75)

With the use of the rule

$$\mathbf{p}\sigma \cdot \mathbf{n}\sigma + \mathbf{n}\sigma \cdot \mathbf{p}\sigma = 2\mathbf{pn} \tag{E.1}$$

the term in (75) can be modified as follows:

$$\begin{aligned} & u^\dagger(\mathbf{p},\lambda\mathbf{n})\mathbf{n\Sigma}u(\mathbf{p},\lambda\mathbf{n}) \\ &= \frac{1}{2N}\phi^\dagger_{\lambda\mathbf{n}}\left(\mathbf{n}\sigma + \frac{\mathbf{p}\sigma\cdot\mathbf{n}\sigma\cdot\mathbf{p}\sigma}{(p_0+m)^2}\right)\phi_{\lambda\mathbf{n}} \\ &= \frac{1}{2N}\phi^\dagger_{\lambda\mathbf{n}}\left(\mathbf{n}\sigma + \frac{\mathbf{p}\sigma\cdot(-\mathbf{p}\sigma\cdot\mathbf{n}\sigma + 2\mathbf{pn})}{(p_0+m)^2}\right)\phi_{\lambda\mathbf{n}} \\ &= \frac{1}{2N}\phi^\dagger_{\lambda\mathbf{n}}\left(\mathbf{n}\sigma\left(\mathbf{1} - \frac{\mathbf{p}^2}{(p_0+m)^2}\right) + \frac{2\mathbf{pn}\cdot\mathbf{p}\sigma}{(p_0+m)^2}\right)\phi_{\lambda\mathbf{n}} \\ &= \frac{1}{2p_0}\phi^\dagger_{\lambda\mathbf{n}}\left(m\cdot\mathbf{n\Sigma} + \frac{\mathbf{pn}\cdot\mathbf{p}\sigma}{p_0+m}\right)\phi_{\lambda\mathbf{n}}\,. \end{aligned} \tag{E.2}$$

Since

$$\left|\phi^\dagger_{\lambda\mathbf{n}}\mathbf{n}\sigma\phi_{\lambda\mathbf{n}}\right| = 1\,, \quad \left|\phi^\dagger_{\lambda\mathbf{n}}\mathbf{p}\sigma\phi_{\lambda\mathbf{n}}\right| \le p\,, \quad \mathbf{pn} = p\cos\alpha\,, \tag{E.3}$$

it follows that

$$|u^\dagger(\mathbf{p},\lambda\mathbf{n})\mathbf{n\Sigma}u(\mathbf{p},\lambda\mathbf{n})| \le \frac{1}{2p_0}\left(m + \frac{p^2}{p_0+m}\right) = \frac{1}{2}\,. \tag{E.4}$$

Obviously

$$|u^\dagger(\mathbf{p},\lambda\mathbf{n})\mathbf{n\Sigma}u(\mathbf{p},\lambda\mathbf{n})| = \frac{1}{2} \tag{E.5}$$

only for $\mathbf{p}/p = \pm\mathbf{n}$ or $p = 0$.

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